Formulation of 1D relaxation operator for Dirichlet and Neumann BCs

# 1D Laplacian / Relaxation

The form of the 1D Laplacian is

We may write this in matrix form as where

The boundary values need to be treated carefully for D and N BCs, also while considering CC and node centered data. Let's start with D BCs.

# Dirichlet

For Dirichlet BCs, if we see the solution to

Then we can simply say that and let on the boundary values. This is a simple treatment.

# Neumann

For Neumann BCs, we must determine the value on the boundary for

## Cell centered data

For CC data, we can write

Which solves

and use a ghost cell to enforce the BCs. In order to do this, the RHS must be adjusted so that can stay in this form. must be adjusted to

And now we may write the equation

## Node centered data

For node centered data we can write

Which solves

and use a ghost cell to enforce the BCs. In order to do this, the RHS must be adjusted so that can stay in this form. must be adjusted to

And now we may write the equation